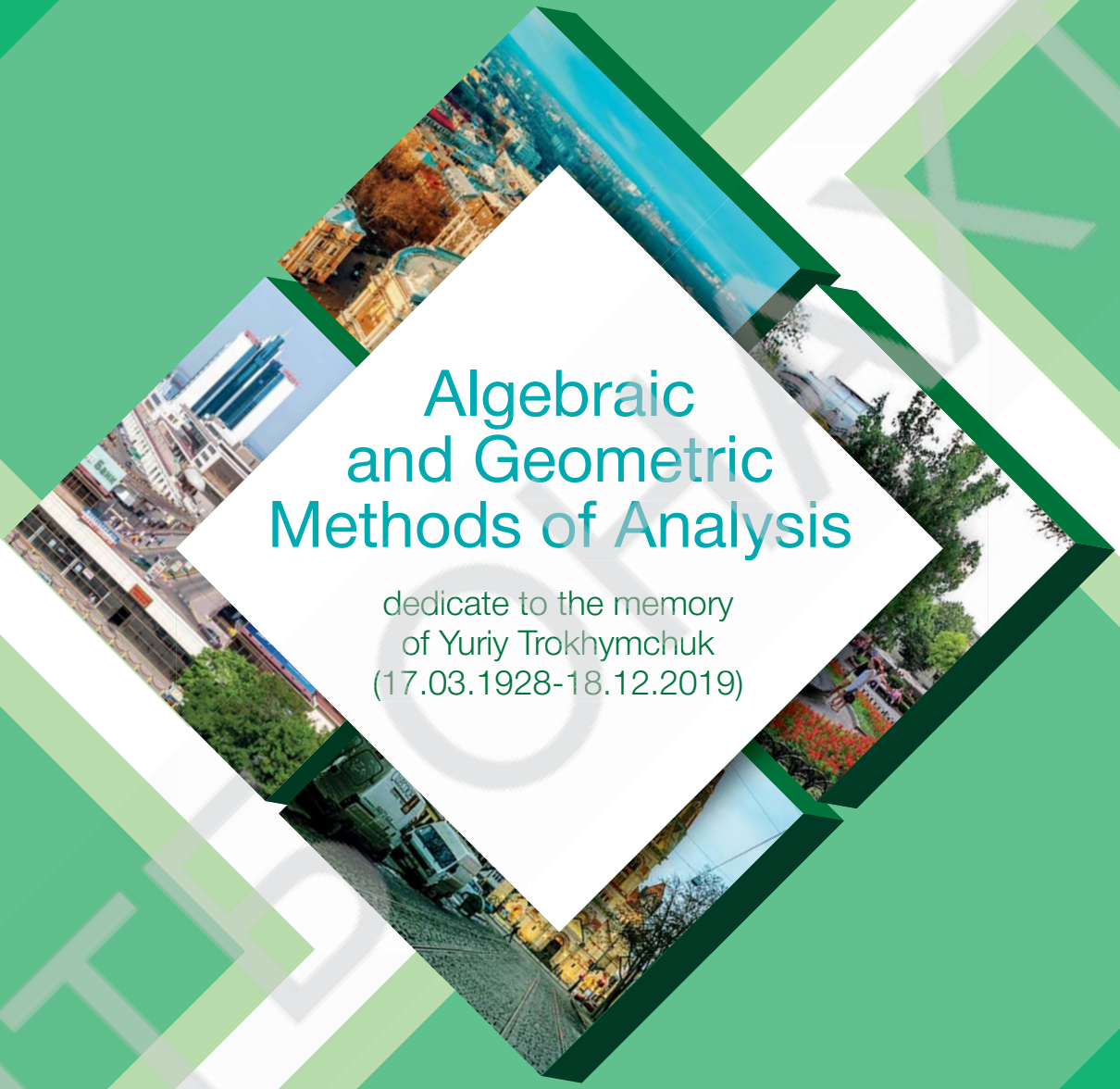
International
Online Conference



Algebraic and Geometric Methods of Analysis

dedicate to the memory
of Yuriy Trokhymchuk
(17.03.1928-18.12.2019)

May 25-28, 2021
Odesa, Ukraine

НТБ ОНМАКТ

List of topics

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

Organizers

- Ministry of Education and Science of Ukraine
- Odesa National Academy of Food Technologies
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- International Geometry Center
- Kyiv Mathematical Society

Scientific Committee

Drozd Yu.
(*Kyiv, Ukraine*)
Maksymenko S.
(*Kyiv, Ukraine*)
Plaksa S.
(*Kyiv, Ukraine*)
Prishlyak A.
(*Kyiv, Ukraine*)

Bakhtin O.
(*Kyiv, Ukraine*)
Balan V.
(*Bucharest, Romania*)
Banakh T.
(*Lviv, Ukraine*)
Borysenko O.
(*Kharkiv, Ukraine*)

Cherevko Ye.
(*Odesa, Ukraine*)
Fedchenko Yu.
(*Odesa, Ukraine*)
Karlova O.
(*Chernivtsi, Ukraine*)
Kiosak V.
(*Odessa, Ukraine*)
Konovenko N.
(*Odessa, Ukraine*)
Lyubashenko V.
(*Kyiv, Ukraine*)
Matsumoto K.
(*Yamagata, Japan*)
Mormul P.
(*Warsaw, Poland*)

Mykhailyuik V.
(*Chernivtsi, Ukraine*)
Plachta L.
(*Krakov, Poland*)
Pokas S.
(*Odessa, Ukraine*)
Sabitov I.
(*Moscow, Russia*)
Savchenko O.
(*Kherson, Ukraine*)
Sergeeva A.
(*Odesa, Ukraine*)
Shelekhov A.
(*Tver, Russia*)
Zarichnyi M.
(*Lviv, Ukraine*)

НТБ ОНАХТ

Administrative committee

- Egorov B., chairman, rector of the ONAFT;
- Povarova N., deputy chairman, Pro-rector for scientific work of the ONAFT;
- Mardar M., Pro-rector for scientific-pedagogical work and international communications of the ONAFT;
- Fedosov S., Director of the International Cooperation Center of the ONAFT;
- Kotlik S., Director of the P.M. Platonov Educational-scientific institute of computer systems and technologies "Industry 4.0";
- Lishchenko N. Dean of faculty of the computer systems and automation ONAFT

Organizing commitee

Cherevko Ye.
Eftekharinasab K.
Fedchenko Yu.
Feshchenko B.
Khohliyk O.
Klishchuk B.
Konovenko N.
Kravchenko A.
Kuznietsova I.
Maksymenko S.
.
Osadchuk E.
Plakosh A.
Prus A.
Sergeeva A.
Soroka Yu.

НТБ ОНАХТ

Projective invariants of linear planar 3-webs

Irina Streltsova
(Astrakhan State University, ul. Tatishcheva 20a, Astrakhan, 414056 Russia)
E-mail: strelzova_i@mail.ru

In this talk we present projective differential invariants of linear planar 3-webs. Linear 3-web on the plane $\mathbb{R}^2(x,y)$ is an unordered set of 3 linear foliations with the condition that leaves of any pair of foliations are transversal to each other. Any such web is defined by the set of 3 solutions $w = w(x,y)$ to the Euler equation (see [1])

$$w_y = ww_x.$$

We will consider actions of the group of projective transformations $SL_3(\mathbb{R})$ of the plane. This actions carries over to the space of solutions of the Euler equation. Representations of the Lie algebra $\mathfrak{sl}_3(\mathbb{R})$ by vector fields are

$$X_A = (a_{13} + (a_{11} - a_{33})x + a_{12}y - a_{32}xy - a_{33}x^2)\partial_x + \\ + (a_{23} + a_{21}x + (a_{22} - a_{33})y - a_{31}xy - a_{32}y^2)\partial_y,$$

where the matrix $A = \|a_{ij}\|_{i,j=1,2,3} \in \mathfrak{sl}_3(\mathbb{R})$.

Proposition 1. *The vector fields*

$$\overline{X}_A = X_A + \lambda_A(w)\partial_w,$$

where

$$\lambda_A(w) = (a_{21} - a_{31}y)w^2 + (a_{11} - a_{22} - a_{31}x + a_{32}y)w + a_{32}x - a_{12},$$

define representations of the Lie algebra $\mathfrak{sl}_3(\mathbb{R})$ on the total space of the bundle

$$\pi' : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^2, \quad \pi'(x,y,w) \mapsto (x,y).$$

Moreover, the vector fields $\overline{X}_A$ are symmetries of the Euler equation.

Linear planar 3-webs are defined by a set of solutions w^1, w^2, w^3 to the Euler equation.

Proposition 2. *The vector fields*

$$\overline{X}_A = X_A + \lambda_A(w^1)\partial_{w^1} + \lambda_A(w^2)\partial_{w^2} + \lambda_A(w^3)\partial_{w^3}$$

define a representation of Lie algebra $\mathfrak{sl}_3(\mathbb{R})$ on the total space of the bundle

$$\pi : \mathbb{R}^2 \times \mathbb{R}^3 \to \mathbb{R}^2, \quad \pi(x,y,w^1,w^2,w^3) \mapsto (x,y).$$

Moreover, the vector fields $\overline{X}_A$ are symmetries of the system of Euler equations

$$w^1_y = ww^1_x, \quad w^2_y = w^2w^2_x, \quad w^3_y = w^3w^3_x. \tag{1}$$

System of equations (1) defines the submanifold

$$E_1 \subset J^1(\pi), \quad E_1 = \{w^1w^1_x - w^1_y = 0, w^2w^2_x - w^2_y = 0, w^3w^3_x - w^3_y = 0\},$$

where $J^1(\pi)$ is the bundle of 1-jets of sections of this bundle. Let $E_k \subset J^k(\pi)$ be a kth prolongation of this manifold.

A rational function I on the manifold E_k is called *a projective differential invariant* of linear 3-webs of order $\leq k$, if $\overline{X}^{(k)}_A(I) = 0$ on E_k for all $A \in \mathfrak{sl}_3(\mathbb{R})$. Here $\overline{X}^{(k)}_A$ is the kth prolongation of the vector field $\overline{X}_A$.

Solving the system of equations $\overline{X}^{(2)}_A(I) = 0$, we get the following result.

Theorem 3. *The field of rational projective differential invariants of order ≤ 2 of linear 3-webs is generated by invariants of order 2*

$$I_{21} = \frac{w^2_{xx}}{w^3_{xx}}, \quad I_{22} = -\frac{(-w^2+w^3)w^1_x + (w^1-w^3)w^2_x - w^3_x(w^1-w^2)}{\sqrt{w^3_{xx}(w^2-w^3)(w^1-w^3)(w^1-w^2)}}, \quad I_{23} = \frac{w^1_{xx}}{w^3_{xx}}.$$

This field separates regular $SL_3(\mathbb{R})$-orbits in E_2.

To describe the field of all projective differential invariants of linear 3-webs, we use the Lie-Tresse theorem (see [2]).

Theorem 4. *The field of rational projective differential invariants of linear 3-webs is generated by the basis invariants I_{21}, I_{22}, I_{23} and the invariant derivations*

$$\nabla_1 = -\frac{(-w^2+w^3)w^1}{w^1w^2_x - w^1w^3_x - w^1_xw^2 + w^1_xw^3 + w^2w^3_x - w^2_xw^3}\frac{d}{dx}$$

$$+\frac{-w^2+w^3}{w^1w^2_x - w^1w^3_x - w^1_xw^2 + w^1_xw^3 + w^2w^3_x - w^2_xw^3}\frac{d}{dy},$$

$$\nabla_2 = -\frac{(w^3-w^1)w^2}{w^1w^2_x - w^1w^3_x - w^1_xw^2 + w^1_xw^3 + w^2w^3_x - w^2_xw^3}\frac{d}{dx}$$

$$+\frac{-w^2+w^3}{w^1w^2_x - w^1w^3_x - w^1_xw^2 + w^1_xw^3 + w^2w^3_x - w^2_xw^3}\frac{d}{dy}.$$

This field separates regular orbits.

References

[1] V.V. Goldberg, V.V. Lychagin. Geodesic webs on a two-dimensional manifold and Euler equations. *Acta Applicandae Mathematicae*, 04/2012; 109(1):5–17. DOI 10.1007/s10440-009-9437-1.

[2] B.S Kruglikov, V.V. Lychagin. Global Lie-Tresse theorem. *Selecta Mathematica*, 02/2016. DOI 10.1007/s00029-015-0220-z.

I. Streltsova *Projective invariants of linear planar 3-webs* **145**

Kh. Sukhorukova *Ultrametric spaces of $*$-measures* **147**

B. M Sultanov *Sweep of surfaces in Galilean space* **148**

Y. Teplitskaya *Maximal distance minimizers. Examples and properties* **149**

M. Tkachuk, S. Plaksa *Analog of Menchov-Trokhimchuk theorem for monogenic functions in three-dimensional commutative algebra* **151**

E. Tosunoğlu, M. E. Kansu *Onto the some dynamic applications via quaternions* **153**

R. M. Tudoran *On dynamical systems with a prescribed globally bp-attracting set* **154**

T. P. Mokritskaya, A. V. Tushev *On certain fractal-based estimations of subsidence volume* **155**

Jun Ueki *Modular knots obey the Chebotarev law* **156**

J. Vielma, A. Guale *The Terence Tao set and the Collatz conjecture* **159**

V. Dilnyi, A. Vinskovska *Conformal mappings in Hardy-type spaces* **160**

I. Vlasenko *On the equivalency classes of weakly conjugated inner mappings* **161**

L. Vyhivska *Extremal problem for non-overlapping domains with free poles* **162**

N. Zorii *Inner harmonic measure for the fractional Laplacian* **163**

П. Задерей, М. Гаєвський *Мультиплікатори в просторах Харді та пов'язаних з ними просторах* **164**

М. І. Піструіл, І. М. Курбатова *Про деякі закономірності квазі-геодезичних відображень узагальнено-рекурентних просторів* **166**

С. М. Покась, И. И. Белокобыльский *Геодезические отображения римановых пространств второго приближения* **167**

НТБ ОНАХТ