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Semi-lattice of varieties of quasigroups with linearity

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We consider two-placed functions defined on an arbitrary set \( Q \), called carrier, whose domain is \( Q \times Q \) and the functions are called binary operations. Left and right multiplications are defined on the set of all binary operations \( O_2 \) by

\[
(f \oplus g)(x, y) := f(g(x, y), y) \quad \text{and} \quad (f \oplus g)(x, y) := f(x, g(x, y))
\]

respectively. The obtained groupoids \((O_2; \oplus)\) and \((O_2; \oplus)\) are called left and right symmetric monoids. An operation \( f \) is called left (right) invertible if it has an invertible element \( f^L \) (\( f^R \)) in the left (right) symmetric monoid and invertible if both are true. \( f^L \) and \( f^R \) are called left and right divisions. The sequence \( f, f^L, f^R, \ldots \) consists of at most six different operations called parastrophes of \( f \):

\[ a_f(x_1, x_2) = x_3 \quad \sigma \in S_3 := \{ \tau \mid \tau \text{ is a permutation of } \{1, 2, 3\} \} \]

Every parastrophe of an invertible operation is invertible. In other words, the set of all invertible binary operations \( \Delta_2 \) defined on a carrier \( Q \) is parastrophically closed. The group \( Ps(f) := \{ \tau \mid \tau f = f \} \) is a subgroup of \( S_3 \) and called parastrophic symmetry group of \( f \). \( Q; f, f^L, f^R \) is quasigroup, if \( f \) is invertible operation and \( Q \) is its carrier. If, in addition, the operation \( f \) is associative, then the quasigroup is a group, i.e., it has a neutral element and every element has an inverse.

Two operations \( f \) and \( g \) are called isotopic if there exists a triple of bijections \((\delta, \nu, \gamma)\) called an isotopism such that \( f(x, y) := \gamma g(\delta^{-1} x, \nu^{-1} y) \) for all \( x, y \in Q \). An isotope of a group is called a group isotope. A class of quasigroups is called a variety if it described by identities.

In most cases solving a problem for some set \( \Phi \) of invertible functions, we solve it for all parastrophes of functions from \( \Phi \). That is why \( \Phi \) is supposed to be parastrophically closed. Very often it is difficult or even impossible to find the general solution of a problem for all operations from \( \Phi \).

**Partition problem:** Find a partition of the given parastrophically closed set of invertible functions into parastrophically closed subsets with respect to the given property.

The second author \[1\] solved this problem for group isotopes with respect to parastrophic symmetry group. Here we consider a partition of group isotopes on an arbitrary carrier with respect to the property of linearity. Let \((Q; \cdot)\) be a group isotope and 0 be an arbitrary element of \( Q \), then the right part of the formula \( x \cdot y = ax + a + \beta y \) is called a 0-canonical decomposition, if \((Q; +, 0)\) is a group and \(a0 = \beta 0 = 0\). An arbitrary element \( b \) uniquely defines \( b \)-canonical decomposition of an arbitrary group isotope \[2\].

In this case: the element 0 is a defining element; \((Q; +)\) is a decomposition group; \(a\) is its free member; \(\alpha\) is its left, i.e., 2-coefficient; \(\beta\) is its right, i.e., 1-coefficient; \(J\alpha\beta^{-1}\) is its middle, i.e., 3-coefficient.

An isotope of an arbitrary group with a canonical decomposition \( x \cdot y = ax + a + \beta y \) is called

- (strictly) \( i \)-linear, if the \( i \)-coefficient is an automorphism of the decomposition group, where \( i = 1, 2, 3 \) (another coefficients are neither automorphism nor anti-automorphism);
- (strictly) \( i \)-alinear, if the \( i \)-coefficient is an anti-automorphism of the decomposition group, where \( i = 1, 2, 3 \) (another coefficients are neither automorphism nor anti-automorphism);
- semi-linear or one-sided linear, if it is \( i \)-linear for some \( i = 1, 2, 3 \);
• **semi-central or one-sided central**, if it is semi-linear and its decomposition group is commutative;
• **central** if it is linear and its decomposition group is commutative;
• **semi-alinear or one-sided alinear**, if it is $i$-alinear for some $i = 1, 2, 3$;
• **$ij$-linear**, if it is $i$-linear and $j$-linear; **linear**, if it is $i$-linear and $j$-linear for some $i \neq j$;
• **alinear**, if it is $i$-alinear for all $i, j \in \{1, 2, 3\}$.
• **anti-linear**, if all its coefficients are neither automorphism nor anti-automorphism.

**Theorem 1.** All group isotope operations on an arbitrary carrier is parted into the following parastrophically closed blocks: 1) strictly semi-linear operations whose decomposition groups are not commutative; 2) strictly semi-alinear operations whose decomposition groups are not commutative; 3) linear operations whose decomposition groups are not commutative; 4) alinear operations whose decomposition groups are not commutative; 5) strictly semi-central operations; 6) central operations; 7) anti-linear operations.

**Theorem 2.** Let $i \in \{1, 2, 3\}$ and $\sigma \in S_3$. If a group isotope is $i$-linear ($i$-alinear), then its $\sigma$-parastrophe is $i\sigma^{-1}$-linear ($i\sigma^{-1}$-alinear).

Quasigroups with linearity form 14 varieties: 1) three pairwise parastrophic varieties of one-sided linear quasigroups: left $\mathcal{L}_L$, right $\mathcal{L}_R$ and middle $\mathcal{L}_M$; 2) three pairwise parastrophic varieties of one-sided alinear quasigroups: left $\mathcal{L}_{al}$, right $\mathcal{L}_{ar}$ and middle $\mathcal{L}_{am}$; 3) three pairwise parastrophic varieties of linear quasigroups: left-right linear $\mathcal{L}_{lr}$ (consequently they are middle alinear), left-middle linear $\mathcal{L}_{lm}$ (consequently they are right alinear) and right-middle $\mathcal{L}_{rm}$ linear (consequently they are left alinear); 4) a variety consisting of all left-right-middle alinear quasigroups $\mathcal{L}_a$; 5) three pairwise parastrophic varieties of one-sided central quasigroups: left $\mathcal{L}_{lc}$, right $\mathcal{L}_{rc}$ and middle $\mathcal{L}_{mc}$; and 6) the variety of all central quasigroup $\mathcal{L}_c$. The varieties form the following semi-lattice.

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